Analysis of Multiple-Strip Discontinuity in a Rectangular Waveguide

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Abstract —**The paper considers the problem of an arbitrary number of thin, transverse, and metallic strips located in the cross section of a rectangular waveguide. The strips can either be inductive or capacitive and are arbitrarily located. Equivalence theorem is utilized, in conjunction with appropriate boundary conditions, to set up an integral equation which is then solved using the method of moments. The method is applied to a number of test problems and the numerical results show good agreement with the available data.**

I. Introduction

A PARTICULARLY important problem in electromagnetic theory is the analysis of obstacles and discontinuities in a waveguide. Several methods of solution have been developed and some of these may be found in standard texts [1], [2]. Although the rigorous analytical techniques give an insight into the finer points of the phenomena involved, the working engineer is generally interested only in the numerical results. Therefore, a recent trend is to utilize numerical mode-matching and moment methods, which has been made possible by the development of powerful digital computers.

This paper considers the problem of a number of thin, transverse, and metallic strips which are arbitrarily located in the cross section of a rectangular waveguide. The related problem of double apertures or double strips has been analyzed by Lewin [2, chs. 6, 7] using the singular-integral equation method. Chang and Khan [3] have used the variational theory to study the problem of coupling between two inductive strips in a rectangular waveguide. Recently, Auda and Harrington [4] have presented a moment procedure for the general problem of inductive obstacles in a rectangular waveguide. In their method, each of the posts is approximated by a finite number of current carrying strips. The waveguide Green's function is then used to express the field produced by these currents. In order to improve the convergence, they express the dynamic Green's function in terms of the corresponding static Green's function and the resulting integrals are evaluated numerically. Although the method can, in principle, be extended to include the capacitive obstacles, the analytical and numerical problems in that case are expected to increase further because the obstacle is no longer uniform in the direction of electric lines of force.

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In this paper, we present an alternative moment procedure which is straightforward and can be applied to the problem of multiple strip discontinuities in a rectangular waveguide. The method is based upon the generalized network formulation for aperture problems [5] and can handle both the inductive and capacitive strips of arbitrary number and width.

II. Formulation

Fig. 1 shows the problem under study and defines the coordinates and parameters used. Here, we consider p perfectly conducting strips of widths $w_1, w_2, \cdots, w_p$ located at the plane $z = 0$. The strips are infinitesimally thin and can be either inductive (Fig. 1(a)) or capacitive (Fig. 1(b)). The waveguide is considered to be lossless and supports only the dominant TE_{10} propagating mode. An electromagnetic source is assumed to be located in the region $z < 0$.

As a first step, the equivalence theorem [1, sec. 1.7] is utilized to divide the problem into two separate parts (Fig. 2). The apertures $A_1, A_2, \cdots, A_q$ are closed with electric conductors and an equivalent surface magnetic current $+\overline{M}(x, y)$ is placed at $z = 0^-$ in the region of apertures. It is given by

$$\overline{M} = \hat{z} \times \overline{E} \tag{1}$$

where $\overline{E}$ is the electric field in the apertures of the original problem and $\hat{z}$ is the unit normal. In order to ensure the continuity of electric field at $z = 0$, a current $-\overline{M}(x, y)$ is placed at $z = 0^+$. The electromagnetic field in the region $z < 0$ (region 1) is now due to the impressed sources and the current $+\overline{M}$ at $z = 0^-$ while in the region $z > 0$ (region 2), it is due to the current $-\overline{M}$ at $z = 0^+$. In both cases, the field is evaluated with the apertures short-circuited.

Enforcing the boundary condition that the transverse (to z-axis) magnetic field is continuous across the apertures, we obtain

$$\overline{H}_t^{(1)}(\overline{M}) + 2\overline{H}_t^{\text{inc}} = \overline{H}_t^{(2)}(-\overline{M}) \qquad \text{on } A \tag{2}$$

where

$$A = \bigcup_{t=1}^{q} A_t \tag{3}$$

and $\cup$ denotes the union operator. In (2), the transverse magnetic field produced in region 1 by the current $+\overline{M}$ is

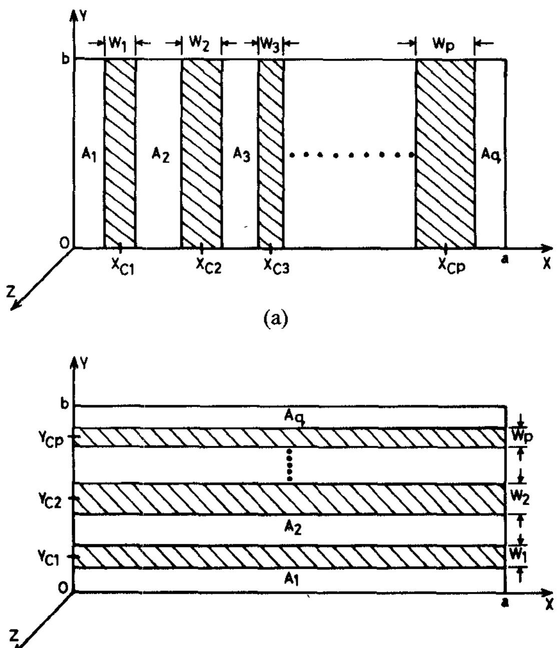

Fig. 1. Geometry of the problem: (a) inductive strips, (b) capacitive strips.

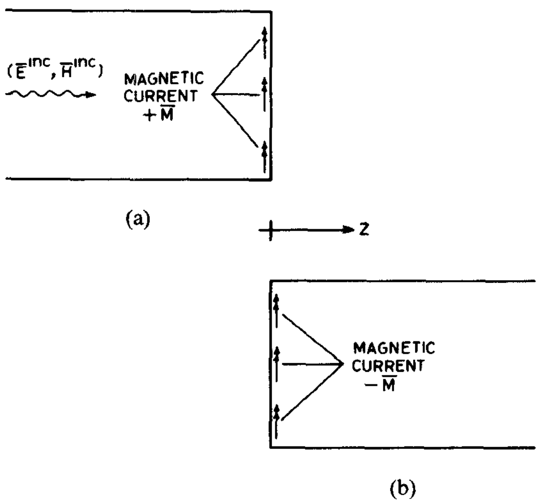

Fig. 2. Equivalent problems: (a) model valid in the region $z<0$, (b) model valid in the region $z>0$.

denoted $\overline{H}_t^{(1)}(\overline{M})$, that produced in region 2 by $-\overline{M}$ is denoted $\overline{H}_t^{(2)}(-\overline{M})$, and the incident field due to the impressed sources is denoted $\overline{H}_t^{\text{inc}}$.

Using the linearity of $\overline{H}_t$ operator and the fact that the two regions are identical, (2) can be rewritten as

$$\overline{H}_t(\overline{M})+\overline{H}_t^{\text{inc}}=0 \qquad \text{on } A. \tag{4}$$

The operator equation (4) can be solved numerically using the method of moments. More specifically, we define a set of expansion functions $\{\overline{M}_j\}$ and a set of testing functions $\{\overline{W}_i\}$ in A and express the current $\overline{M}$ by the superposition

$$\overline{M}=\sum_{j=1}^{N} V_j\overline{M}_j \tag{5}$$

where V_j are the unknown complex coefficients.

Following the procedure outlined in [5], (4) can now be reduced to matrix form

$$[Y]\vec{V}=\vec{I} \tag{6}$$

where

$$[Y]=\left[\langle -\overline{W}_i, \overline{H}_t(\overline{M}_j)\rangle\right]_{N\times N} \tag{7}$$

$$\vec{I}=\left[\langle \overline{W}_i, \overline{H}_t^{\text{inc}}\rangle\right]_{N\times 1} \tag{8}$$

$$\vec{V}=\left[V_j\right]_{N\times 1}. \tag{9}$$

Solution of the system of equations (6) determines the equivalent magnetic current $\overline{M}$.

A. Evaluation of Matrix Elements Y_{ij}

The transverse components of the fields produced by a single expansion function $+\overline{M}_j$ can be written in the modal form as [1, sec. 5.6]

$$\overline{E}_t(\overline{M}_j)=\sum_{s=1}^{K} C_{js}e^{\gamma_s z}\,\overline{e}_s \tag{10}$$

$$\overline{H}_t(\overline{M}_j)=-\sum_{s=1}^{K} C_{js}Y_s e^{\gamma_s z}\,\hat{z}\times\overline{e}_s \tag{11}$$

where K is the number of modes used to approximate the fields, C_{js} are the modal amplitudes, $\overline{e}_s$ are the normalized modal vectors, γ_s are the modal propagation constants, and Y_s are the modal characteristic admittances.

Substituting (10) in (1) and making use of the orthogonality of modal vectors, one can easily show that

$$C_{js}=\iint \overline{M}_j\cdot\hat{z}\times\overline{e}_s\,ds. \tag{12}$$

An element of the matrix $[Y]$ is now obtained in a straightforward manner by combining (7) and (11) and is given by

$$Y_{ij}=\sum_{s=1}^{K} B_{is}Y_sC_{js} \tag{13}$$

where the coefficients B_{is} are given by (12) with $\overline{M}_j$ replaced by $\overline{W}_i$.

B. Evaluation of Excitation Vector $\vec{I}$

Assuming that only the dominant mode of unit amplitude is incident at $z=0$, the transverse magnetic field in the incident wave can be written as

$$\overline{H}_t^{\text{inc}}=Y_o e^{-\gamma_o z}\,\hat{z}\times\overline{e}_o \tag{14}$$

where the subscript o denotes the dominant mode.

Substituting (14) in (8), an element of excitation vector is obtained as

$$I_i=Y_oB_{io}. \tag{15}$$

III. Determination of the Equivalent Circuit

From an engineering viewpoint, it is always desirable to characterize a discontinuity by an electrical network containing lumped elements. The class of discontinuities con-

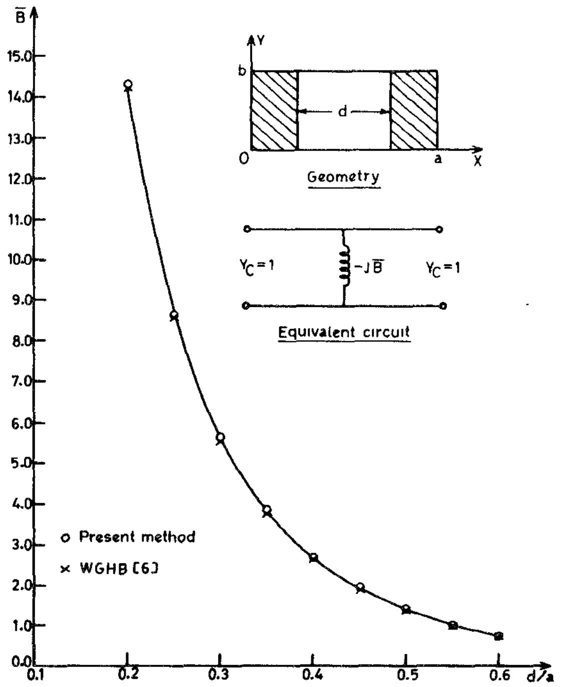

Fig. 3. The normalized susceptance of the symmetrical inductive diaphragm ($a/\lambda = 0.8$).

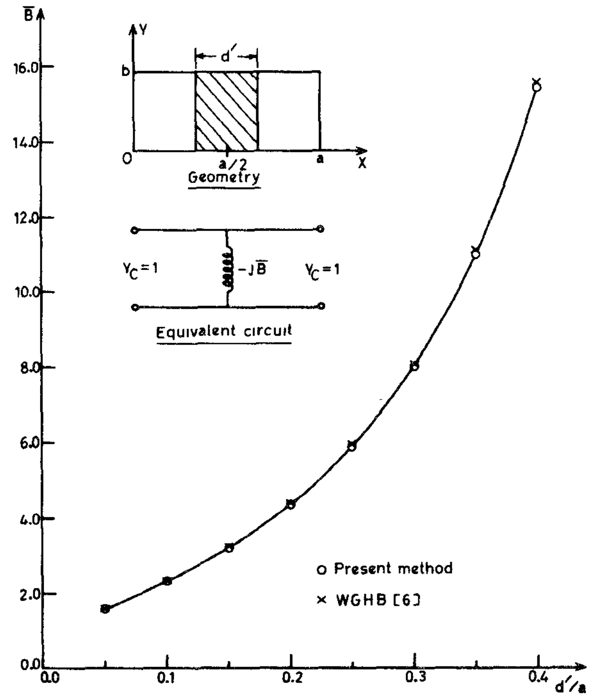

Fig. 4. The normalized susceptance of the symmetrical inductive strip ($a/\lambda = 0.8$).

sidered here can be represented simply by a normalized shunt susceptance $j\bar{B}$ across a transmission line of unit characteristic admittance. It is given by [1, sec. 8.2]

$$j\bar{B} = -\frac{2\Gamma_o}{1+\Gamma_o} \tag{16}$$

where Γ_o is the reflection coefficient of the dominant mode, evaluated at $z = 0$, and is given by

$$\Gamma_o = -1 + \sum_{j=1}^{N} V_j C_{jo}. \tag{17}$$

The junction can now be characterized by the scattering matrix

$$S = \begin{bmatrix} \Gamma_o & 1+\Gamma_o \\ 1+\Gamma_o & \Gamma_o \end{bmatrix}. \tag{18}$$

IV. Results and Discussion

The method presented has been applied to a number of problems and the results compared with the available data. A Galerkin procedure has been used with a rooftop function utilized for expansion as well as testing. Some of the problems considered and the results obtained are shown in Figs. 3–6. In all the cases, the moment procedure is found to converge monotonically as shown in Fig. 7 for a capacitive diaphragm.

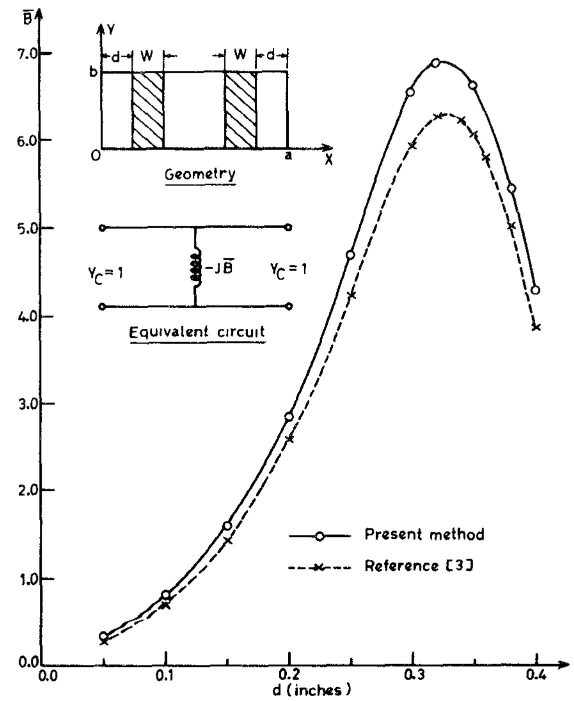

Fig. 5. The normalized susceptance of the double inductive strip ($a = 0.9$ in, $b = 0.4$ in, $w = 0.05$ in, $f = 8.25$ GHz).

It is seen that for the cases of inductive and capacitive diaphragms and the symmetrical inductive strip, the results from the present method agree very well with the data given in the *Waveguide Handbook* (WGHB) [6]. However, for the case of two coupled-inductive strips, the moment

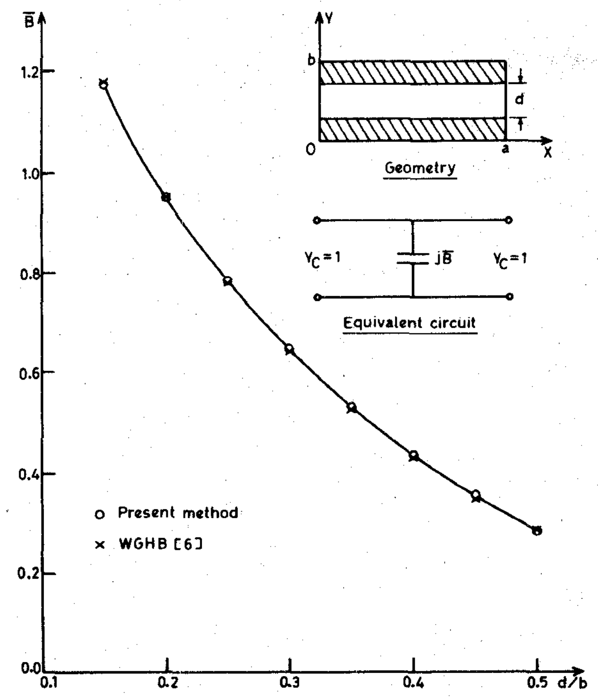

Fig. 6. The normalized susceptance of the symmetrical capacitive diaphragm ($b/\lambda_g = 0.2$).

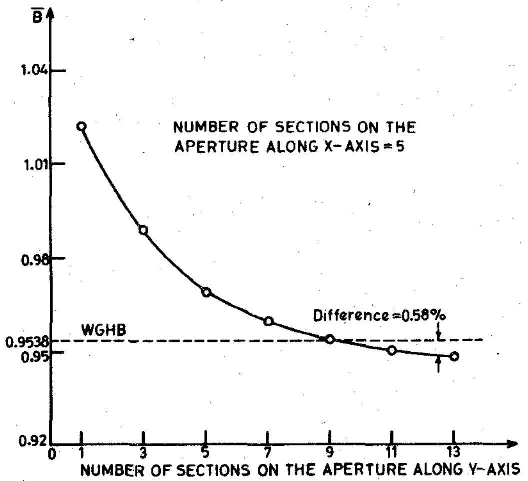

Fig. 7. The convergence of the moment procedure for the symmetrical capacitive diaphragm ($b/\lambda_g = 0.2$, $d/b = 0.2$).

procedure predicts slightly higher values of susceptance than those calculated by Chang and Khan [3]. This is because they assume the current to be constant over the strip-width, which is not strictly true even for narrow strips. As a matter of fact, the difference is found to be as large as 20 percent when the strips are near the narrow walls (large rate of change of electric field), and reduces to about 9 percent when they are near the center of the waveguide.

Some general remarks regarding the numerical procedure are in order here.

In the case of inductive strips, the discontinuity is uniform along the y-axis, and since the electromagnetic energy

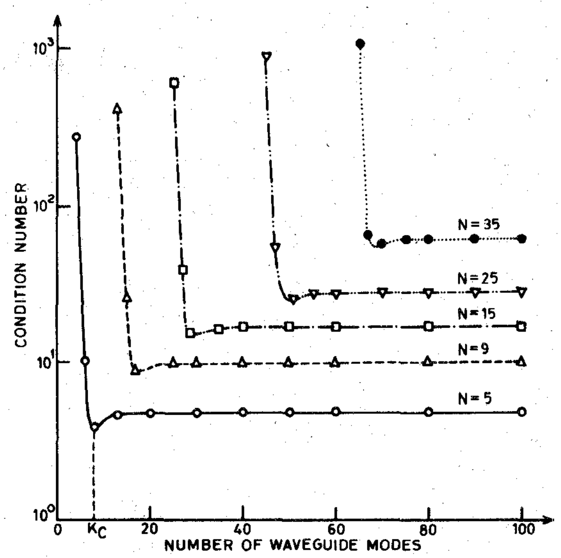

Fig. 8. Condition number of the moment matrix as a function of waveguide modes for a symmetrical inductive diaphragm ($a/\lambda = 0.8$, $d/a = 0.5$).

is incident in the dominant TE_{10} mode, only TE_{mo} modes are excited at $z = 0$. Thus, the equivalent surface magnetic current has only the x-component M^x, which does not vary with y. Consequently, it is the number of sections along the x-axis that affect the convergence of the solution and, moreover, only TE_{mo} modes need be included in the summation of (13).

In the case of capacitive strips, on the other hand, the discontinuity is nonuniform along the y-axis. Therefore, a number of subsections must be used along the y-axis (Fig. 7) and both TE and TM modes must be included.

Further, the numerical solution to the matrix equation (6) is found to exhibit the phenomenon of relative convergence [7], [8]. Thus, the solution may converge to an incorrect value if for a given number of expansion functions, the number of modes used is less than a certain critical value K_c. A convenient way to find the value of K_c is to plot the condition number [9, sec. 5.1] of the moment matrix as shown in Fig. 8. It is seen that for a given number of expansion functions, the condition number settles around a minimum value which is obtained at $K = K_c$. Thus, one should always choose $K > K_c$.

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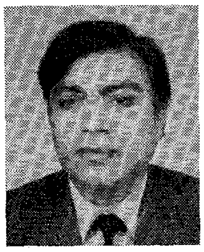

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